

# UNCLASSIFIED

AD NUMBER
AD907150
NEW LIMITATION CHANGE
TO Approved for public release, distribution unlimited
FROM Distribution authorized to U.S. Gov't. agencies only; Test and Evaluation; Jan 1973. Other requests shall be referred to U.S. Army Electronics Command, Fort Monmouth, NJ 07703.
AUTHORITY
USAEC ltr, 4 Mar 1974

THIS PAGE IS UNCLASSIFIED

AD

Research and Development Technical Report  
ECOM-0292-5

# ANALYTIC MATHEMATICAL MODELS OF TACTICAL MILITARY COMMUNICATIONS CHANNELS

QUARTERLY REPORT  
JANUARY, 1973

R.T. CHIEN  
A.H. HADDAD  
C.L. CHEN

## DISTRIBUTION STATEMENT

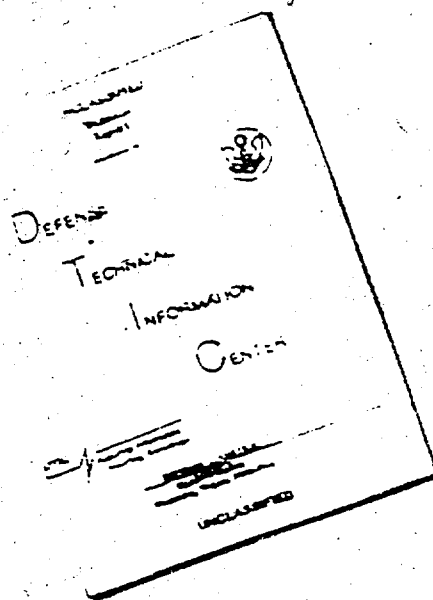
Distribution limited to U.S. Government agencies only;  
Test and Evaluation; January 73. Other requests for  
this document must be referred to Commanding General  
U.S. Army Electronics Command, ATTN: AMSEL-NL-R-2,  
Fort Monmouth, New Jersey 07703

# ECOM

UNITED STATES ARMY ELECTRONICS COMMAND • FORT MONMOUTH, N.J.

Contract DAAB07-71-C-0292  
Coordinated Science Laboratory  
University of Illinois  
Urbana, Illinois 61801

# DISCLAIMER NOTICE



THIS DOCUMENT IS BEST  
QUALITY AVAILABLE. THE COPY  
FURNISHED TO DTIC CONTAINED  
A SIGNIFICANT NUMBER OF  
PAGES WHICH DO NOT  
REPRODUCE LEGIBLY.

REPRODUCED FROM  
BEST AVAILABLE COPY

## NOTICES

### Disclaimers

The findings in this report are not to be construed as an official Department of the Army position, unless so designated by other authorized documents.

The citation of trade names and names of manufacturers in this report is not to be construed as official Government indorsement or approval of commercial products or services referenced herein.

### Disposition

Destroy this report when it is no longer needed. Do not return it to the originator.

TR ECOM-0292-5  
January 1973

Reports Control Symbol  
OSD-1366

ANALYTIC MATHEMATICAL MODELS OF TACTICAL  
MILITARY COMMUNICATIONS CHANNELS

FIFTH QUARTERLY PROGRESS REPORT

1 July 1972 to 30 September 1972

Contract No. DAAB07-71-C-0292  
DA Project No. 1S6.62703.A326.06.07

DISTRIBUTION STATEMENT

Distribution limited to U.S. Government agencies  
only; Test and Evaluation; January 4, 1973. Other  
requests for this document must be referred to  
Commanding General, U.S. Army Electronics Command,  
ATTN: AMSEL-NL-R-2, Fort Monmouth, New Jersey 07703

Prepared by

R. T. Chien  
A. H. Haddad  
C. L. Chen

Coordinated Science Laboratory  
University of Illinois at Urbana-Champaign  
Urbana, Illinois 61801

For

U. S. ARMY ELECTRONICS COMMAND, FORT MONMOUTH, N. J.

# ABSTRACT

Burst distribution was derived using the generating function. However, this method is only practical for small values of  $K$ . A special case for  $K = 2$  was carried out. The result agrees with that derived directly from the gap distributions. Numerical results for  $K = 2$  have been obtained for the VHF channel.

Product codes were discussed and the probability of error for the product code was derived in terms of  $P_e(m,n)$ , the probability that  $m$  errors occurred in  $n$  bits with each bit  $l$  positions apart from the next bit.

## TABLE OF CONTENTS

<u>Section</u>		<u>Page</u>
1	SUMMARY	1
2	MARKOV GAP MODELS	3
	2.1 BURST DISTRIBUTIONS	3
	2.2 SPECIAL CASES, $K = 2$	5
	2.3 NUMERICAL RESULTS FOR THE VHF CHANNEL	7
3	CODE EVALUATION	11
	3.1 PRODUCT CODES	11

# LIST OF ILLUSTRATIONS

<u>Figure</u>		<u>Page</u>
1	The burst distribution $P_b(m)$ for the VHF channel for $K = 2$	9



## SECTION 1

### SUMMARY

Burst distribution was derived using the generating function. However, this method is only practical for small values of  $K$ . A special case for  $K = 2$  was carried out. The result agrees with that derived directly from the gap distributions. Numerical results for  $K = 2$  have been obtained for the VHF channel.

Product codes were discussed and the probability of error for the product code was derived in terms of  $P_d(m,n)$ , the probability that  $m$  errors occurred in  $n$  bits with each bit  $l$  positions apart from the next bit.

## SECTION 2

### MARKOV GAP MODELS

#### 2.1. BURST DISTRIBUTIONS

In the Fourth Quarterly Progress Report [1] recursive expressions were derived for the burst distributions of the gap Markov model using the definition of a burst as a sequence of bits starting and ending in an error and separated from neighboring bursts by at least K error free bits. The resulting expression for the burst rate  $p_b(m)$  is

$$p_b(m) = \sum_{i=0}^{K-1} f_i(m) P(K|i) \quad , \quad m \geq 2 \quad (1)$$

where  $P(m|n)$  is the conditional gap distribution and where the  $f_i(m)$  satisfy the recursive relations

$$f_i(m) = \sum_{j=0}^{K-1} f_j(m-i-1) [P(i|j) - P(i+1|j)] \quad , \quad m \geq i+3 \quad (2)$$

$$f_i(i+2) = \left\{ \sum_{j=K}^{\infty} [P(i|j) - P(i+1|j)] [P(j) - P(j+1)] \right\} / P(K) \quad (3)$$

$$f_i(m) = 0, \quad m < i+2 \quad (4)$$

and where  $P(m)$  is the unconditional gap distribution.

The relation given above may be implemented directly using the gap distributions of the channel as inputs, and the results may then be compared with the burst distributions obtained directly from the data to check the validity of the model. This procedure was performed for solid bursts ( $K=1$ ) for both the VHF and the Tropo channels with excellent results [1]. The results for higher values of the parameter K are being processed. Alternatively, the generating function method may be used to solve the difference equation (2) with (3) and (4) as initial conditions. However, such a solution is practical only for low values of K unless the number of states of the model is reduced by using  $P(m|n_1 \leq n \leq n_2)$  instead of  $P(m|n)$  for all  $n_1 \leq n \leq n_2$ . Let  $\tilde{f}_i(z)$  be defined by

$$\tilde{f}_i(z) = \sum_{m=0}^{\infty} f_{i-1}(m+i+1) z^{m+i} \quad , \quad i=1,2,\dots,K \quad (5)$$

If (2) is substituted in (5) then

$$\begin{aligned}\tilde{f}_i(z) &= f_{i-1}(i+1)z^i + \sum_{m=1}^{\infty} \sum_{j=0}^{K-1} f_j[m+i+1-(i-1)-1]z^{m+i} [P(i-1|j)- \\ &\quad P(i|j)] \\ &= f_{i-1}(i+1)z^i + \sum_{j=1}^K \sum_{m=1}^{\infty} F_{ij} f_{j-1}(m+1)z^{m+i}\end{aligned}\quad (6)$$

where we have defined

$$F_{ij} \triangleq [P(i-1|j-1)-P(i|j-1)] \quad , \quad i, j = 1, 2, \dots, K \quad (7)$$

Since  $f_{j-1}(m+1) = 0$  for  $(m+1) < (j-1+2)$ , (6) may be reduced to

$$\begin{aligned}\tilde{f}_i(z) &= f_{i-1}(i+1)z^i + z^i \sum_{j=1}^K F_{ij} \sum_{m=j}^{\infty} f_{j-1}(m+1)z^m \\ &= f_{i-1}(i+1)z^i + z^i \sum_{j=1}^K F_{ij} \sum_{n=0}^{\infty} f_{j-1}(n+j+1)z^{n+j} \quad , \quad \text{for } n=m-j \\ &= z^i \{f_{i-1}(i+1) + \sum_{j=1}^K F_{ij} \tilde{f}_j(z)\}\end{aligned}\quad (8)$$

which may also be written compactly as

$$[\Lambda(z) - F] \tilde{\underline{f}}(z) = \underline{a} \quad (9)$$

where  $\Lambda(z)$  is diagonal matrix whose elements are  $(\frac{1}{z}, \frac{1}{z^2}, \dots, \frac{1}{z^K})$ ,

$F$  is a  $K \times K$  matrix whose elements are  $F_{ij}$ , and the  $K$ -dimensional vectors  $\tilde{\underline{f}}(z)$  and  $\underline{a}$  have elements  $\tilde{f}_i(z)$  and  $a_i$  respectively,

$$a_i = f_{i-1}(i+1) = \frac{1}{P(K)} \{P(i-1)-P(i) - \sum_{j=1}^K F_{ij} [P(j-1)-P(j)]\},$$

$$i = 1, 2, \dots, K. \quad (10)$$

Equation (9) may now be solved for  $\tilde{\underline{f}}(z)$  and then the  $f_i(m)$  may be obtained by using (5) which finally yields the burst rate  $p_b(m)^i$  for  $m \geq 2$ . If  $F$  is nonsingular (which means that the conditional gap distributions are distinct) then the numerator of

$$\det|\Lambda(z) - F|$$

is a polynomial in  $z$  of degree  $\frac{K(K+1)}{2}$ . If its roots  $\{\beta_i\}$  are also assumed to be distinct then the general form of the burst rate is given by

$$p_b(m) = \sum_{i=1}^{\frac{K(K+1)}{2}} K_i \beta_i^m \quad m \geq 2 \quad (11)$$

The impracticality of this form for large values of  $K$  is immediately apparent as the number of terms increases rapidly. However, for larger values of  $K$  the conditional gap distributions are not available distinctly, and therefore the elements of  $F$  are no longer distinct. In that case the degree of the polynomial in  $z$  is reduced, and consequently the general form (11) has a smaller number of terms. The case where  $K = 1$  has already been considered. The case where  $K = 2$  will be discussed next both exactly and approximately using a reduced number of states.

## 2.2. SPECIAL CASE, $K = 2$

The solution (9) for  $K = 2$  may be easily shown to result in the following expressions

$$\tilde{f}_i(z) = z^i \frac{\Delta_i(z)}{\Delta(z)}, \quad i = 1, 2 \quad (12)$$

where

$$\Delta(z) = 1 - zF_{11} - z^2F_{22} + z^3[F_{11}F_{22} - F_{12}F_{21}] \quad (13)$$

$$\Delta_1(z) = a_1 + z^2[a_2F_{12} - a_1F_{22}] \quad (14)$$

$$\Delta_2(z) = a_2 + z[a_1F_{21} - a_2F_{11}] \quad (15)$$

$$a_i = f_{i-1}(i+1) = \frac{1}{P(2)} [p(i-1) - F_{11}p(0) - F_{12}p(1)], \quad i = 1, 2 \quad (16)$$

where

$$p(i) = [P(i) - P(i+1)]$$

are the unconditional gap probabilities. If  $\Delta(z)$  is factored as follows

$$\Delta(z) = \prod_{j=1}^3 (1 - \beta_j z) \quad (17)$$

then the expressions for  $f_i(m)$  may be obtained from (12) and (5), and are

given by the expressions

$$f_i(m+i+2) = \sum_{j=1}^3 A_{ij} \beta_j^{m+1}, \quad i = 0, 1, \quad m \geq 0 \quad (18)$$

where

$$A_{ij} = - \frac{\Delta_{i+1}(\beta_j^{-1})}{\Delta'(\beta_j^{-1})} = \frac{\Delta_{i+1}(\beta_j^{-1})}{F_{11} + 2\beta_j^{-1}F_{22} - 3\beta_j^{-2}(F_{11}F_{22} - F_{12}F_{21})} \quad (19)$$

where  $\Delta'(\beta_j^{-1}) = \frac{d}{dz} \Delta(z) \big|_{z=\beta_j^{-1}}$ .

Therefore, the general expression for the burst rate is given by

$$p_b(2) = f_0(2)P(2|0) \quad (20)$$

$$p_b(m) = \sum_{j=1}^3 \left[ A_{0j} \frac{P(2|0)}{\beta_j} + A_{1j} \frac{P(2|1)}{\beta_j^2} \right] \beta_j^m, \quad m \geq 3 \quad (21)$$

In order to simplify these expressions and reduce the number of states, the conditional distributions  $P(i|0)$  and  $P(i|1)$  will be assumed to be identical, and will be replaced by  $P(i|0 \leq n \leq 1)$ , i.e.  $P(i|0)$  and  $P(i|1)$  are approximated by

$$P(i|0,1) = \frac{P(i|0)p(0) + P(i|1)p(1)}{1-P(2)} \quad (22)$$

In this case the elements of  $F$  become

$$F_{12} = F_{11} = 1 - P(1|0,1) \quad (23)$$

$$F_{21} = F_{22} = P(1|0,1) - P(2|0,1)$$

Similarly the  $a_i$  becomes

$$a_1 = f_0(2) = \frac{1}{P(2)} \{p(0) - F_{11} [1-P(2)]\} \quad (24)$$

$$a_2 = f_1(3) = \frac{1}{P(2)} \{p(1) - F_{22} [1-P(2)]\}$$

The expression for  $\Delta(z)$  reduces to

$$\Delta(z) = 1 - zF_{11} - z^2F_{22} = (1 - \beta_1 z)(1 - \beta_2 z) \quad (25)$$

with

$$\beta_{1,2} = \frac{1}{2}[F_{11} \pm (F_{11}^2 + 4F_{22})^{\frac{1}{2}}] \quad (26)$$

Finally, the  $f_i(z)$  may be combined since  $P(2|0)$  and  $P(2|1)$  in (21) are replaced by  $P(2|0,1)$ , so that

$$\tilde{f}_1(z) + \tilde{f}_2(z) = \frac{z\Delta_1 + z^2\Delta_2}{\Delta(z)} = z \frac{a_1 + z a_2}{z} \quad (27)$$

From (1) and (5) the expression for  $p_b(m)$  may be derived as follows

$$\begin{aligned} \tilde{f}_1(z) + \tilde{f}_2(z) &= z \sum_{m=0}^{\infty} [f_0(m+2)z^m + f_1(m+2)z^m] \\ &= \frac{z}{P(2|0,1)} \sum_{m=0}^{\infty} z^m p_b(m+2) \end{aligned} \quad (28)$$

which with (27) and (25) yields

$$\begin{aligned} p_b(m+2) &= \left[ \frac{\beta_1 a_1 + a_2}{(\beta_1 - \beta_2)} \beta_1^m - \frac{\beta_2 a_1 + a_2}{(\beta_1 - \beta_2)} \beta_2^m \right] P(2|0,1) \\ &= \frac{P(2|0,1)}{(\beta_1 - \beta_2)} [a_1(\beta_1^{m+1} - \beta_2^{m+1}) + a_2(\beta_1^m - \beta_2^m)], m \geq 0 \end{aligned} \quad (29)$$

The expression (29) for  $m = 0$  agrees with the expression given in (20) if  $P(2|0)$  is replaced by  $P(2|0,1)$ . It is seen though that the number of the exponential terms in (29) has been reduced from three in (21) to two.

### 2.3. NUMERICAL RESULTS FOR THE VHF CHANNEL

Numerical results for  $K = 2$  have been obtained for the VHF channel. In this case the gap data yielded the following distributions,

$$\begin{array}{ll} P(1|0) = 0.6062 & P(1|1) = 0.5944 \\ P(2|0) = 0.4063 & P(2|1) = 0.3884 \\ p(0) = 0.3868 & p(1) = 0.1293 \end{array}$$

Consequently the approximate values of  $F_{11}$  and  $F_{22}$  are

$$F_{11} = 0.3977 \quad F_{22} = 0.2119$$

so that the exponents  $\beta_i$  are given by

$$\beta_1 = 0.7003$$

$$\beta_2 = -0.3026$$

Which when substituted in (29) yields the following expression

$$p_b(m) = (0.07618)(0.7003)^{m-2} - (0.01016)(-0.3026)^{m-2}, \quad m \geq 2.$$

The expression for  $m = 1$  is obtained separately as shown in (1) and should be in exact agreement with the data, as it does not involve the assumptions of the model. The result is

$$p_b(1) = 1 - \frac{P(2|0,1) [1-P(2)]}{P(2)} = 0.7475$$

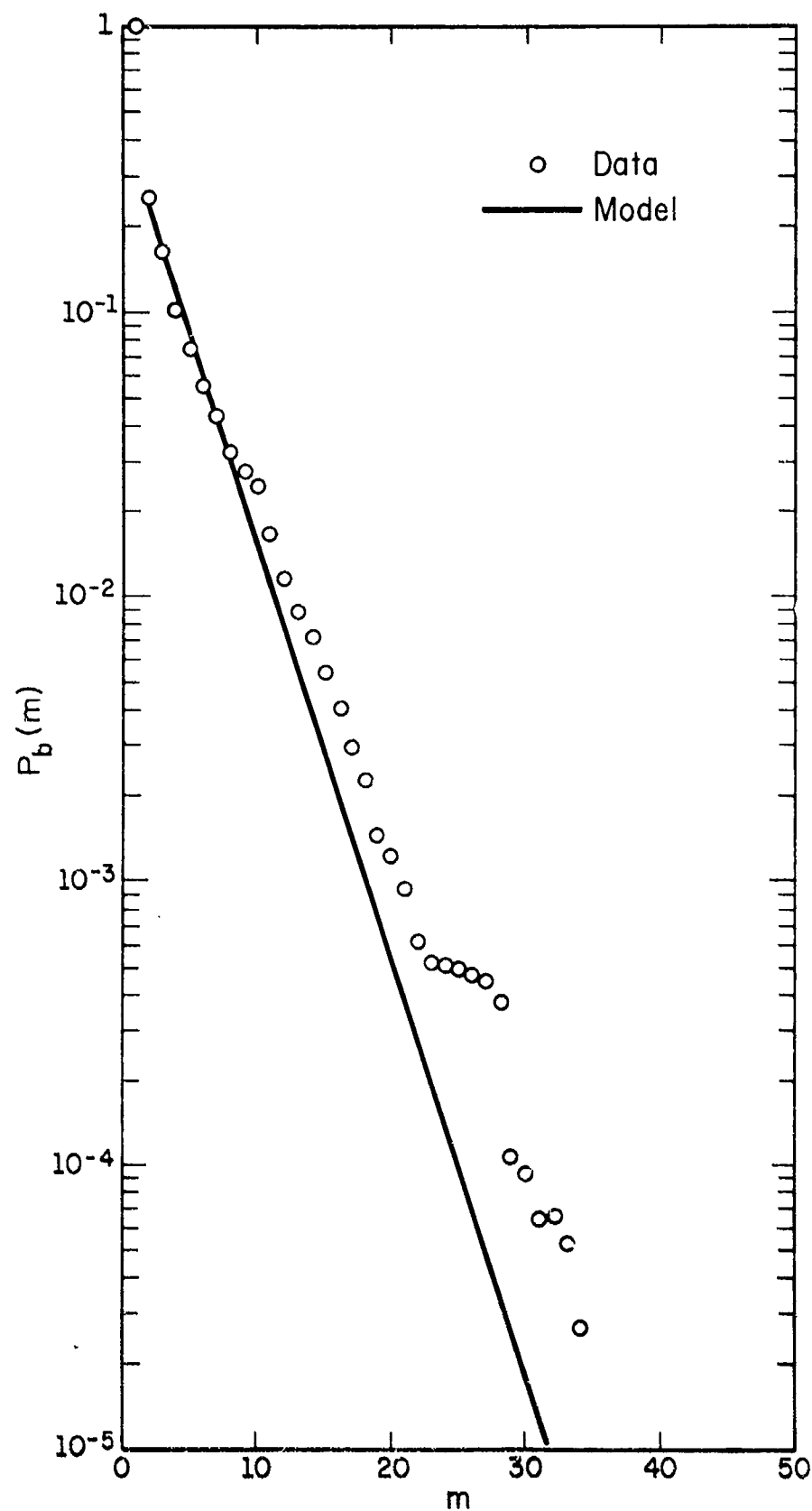
which is identical to the burst rate obtained directly from the data. For the purpose of comparison to the data the expression for the anti-cumulative burst distribution  $P_b(m)$  is obtained

$$P_b(m) = \sum_{k=m}^{\infty} p_b(k) = 0.2542(0.7003)^{m-2} - 0.0077(-0.3026)^{m-2}, \quad m \geq 2$$

Since both the coefficient and the root of the second term are small compared to the first term, its effect is negligible for  $m \geq 5$ , so that an approximate expression for  $P_b(m)$  is given by

$$P_b(m) \approx 0.2542(0.7003)^{m-2}$$

The approximate expression for the burst distribution given from the model is shown in Fig. 1 together with the distribution obtained directly from the data. It is seen that it is in excellent agreement up to  $m = 8$ . For higher values of  $m$  the approximation is still relatively very good in view of the fact that the probabilities involved are very small. The exact expression of (21) is expected to yield a better approximation to the data as it has an additional exponent which would help improve the fit. Definite statements concerning the validity of the model may be made only after the comparison for several values of  $K$  has been performed.



FP-3336

Figure 1. The burst distribution  $P_b(m)$  for the VHF channel for  $K = 2$ .



REFERENCES FOR SECTION 2

- [1] R. T. Chien, F. P. Preparata, A. H. Haddad and C. L. Chen,  
Fourth Quarterly Progress Report, Contract No. DAAB-07-71-C-0292  
for U. S. Army Electronics Command, Fort Monmouth, New Jersey,  
October 1972.

## SECTION 3

### CODE EVALUATION

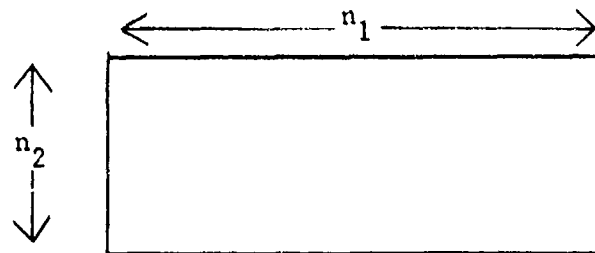
#### 3.1. PRODUCT CODES

Product codes [1] are attractive in many types of data communication systems. They are capable of correcting simultaneously random errors and burst errors. Although their implementation is slightly more complicated than that of interleaved codes, product codes are considerably more powerful than interleaved codes.

The decoding of a product code, first proposed by Elias, can be reduced to the decoding of its subcodes of shorter code length. In this type of decoding, rows are decoded first, then columns and if necessary code words of higher dimensions are decoded. Naturally, majority-logic decoding can be applied to the decoding of the subcodes. Furthermore, it has been shown recently that the information about the decoding of row codes can be carried over to the decoding of the column codes, thus a product code is capable of correcting all the error patterns guaranteed by the minimum distance in addition to many other correctable errors [2].

The performance of a product code on a real channel depends on the decoding scheme used. We will consider only the two-dimensional product codes. Furthermore, a cascade decoding [3] scheme will be used. This decoding scheme can be simply implemented. In addition, the probability of error can be calculated from the statistics of the channel data.

A two-dimensional  $(n_1, n_2, k_1, k_2)$  product code is the direct product of an  $(n_1, k_1)$  row code and an  $(n_2, k_2)$  column code. Let  $t_1$  and  $t_2$  be the error correcting capabilities of the row code and column code, respectively. A code word of the product code can be arranged in a two-dimensional array so that every row forms a code word of the  $(n_1, k_1)$  code and every column forms a code word of the  $(n_2, k_2)$  code. The array is depicted as follows:



Let us assume that a code word is transmitted column by column. At the receiver, rows are decoded according to the decoding scheme for  $(n_1, k_1)$  code. Columns are decoded according to the decoding scheme for  $(n_2, k_2)$  code after all rows have been decoded.

Let  $P_\ell(m,n)$  be the probability that  $m$  errors occurred in  $n$  bits, with each bit  $\ell$  positions apart from the next bit.  $P_\ell(m,n)$  can be obtained directly from the channel data.

The probability of  $m$  errors in a row is by our definition equal to  $P_{n_2}(m, n_1)$ . Since the row code is capable of correcting  $t_1$  errors. The probability of decoding error in a row is

$$\begin{aligned} S &= \Pr\{\text{more than } t_1 \text{ errors in a row}\} \\ &= P_{n_2}(> t_1, n_1) \\ &= \sum_{i=t_1+1}^{n_2} P_{n_2}(i, n_1) \end{aligned} \quad (1)$$

Here the bounded-distance decoding of the row code is assumed. The probability of correctly decoding is

$$\begin{aligned} 1 - S &= \Pr\{\text{at most } t_1 \text{ errors in a row}\} \\ &= P_{n_2}(\leq t_1, n_1) \\ &= \sum_{i=0}^{t_1} P_{n_2}(i, n_1) \end{aligned} \quad (2)$$

Let us assume that the probability of decoding error in a row is made independent of the other row. Thus  $S$  is a constant, independent of row code words. The probability of  $j$  errors in a column is obtained from the binomial distribution

$$P(j, n_2) = \binom{n_2}{j} S^j (1-S)^{n_2-j}. \quad (3)$$

The probability of error for the product code is then

$$\begin{aligned} P_e &\leq \sum_{j=t_2+1}^{n_2} P(j, n_2) \\ &= \sum_{j=t_2+1}^{n_2} \binom{n_2}{j} S^j (1-S)^{n_2-j} \\ &= 1 - \sum_{j=0}^{t_2} \binom{n_2}{j} S^j (1-S)^{n_2-j} \end{aligned} \quad (4)$$

In the next period, several product codes will be evaluated for their performance according to Equation (4).

REFERENCES FOR SECTION 3

- [1] P. Elias, "Error-Free Coding," IRE Trans., IT-4, pp. 29-37, 1954.
- [2] S. M. Reddy, "On Decoding Iterated Codes," IEEE Trans., IT-16, pp. 624-627, September 1970.
- [3] N. Abramson, "Cascade Decoding of Cyclic Product Codes," Technical Report, Department of Electrical Engineering, University of Hawaii, Honolulu, Hawaii, 1967.

### Distribution List

101	Defense Documentation Center ATTN: DDC-TCA Cameron Station (Bldg 5) 10 Alexandria, Virginia 22314	405	DFC, Asst. Sec. of the Army (R&D) ATTN: Asst. for Research Room 3-E-373, The Pentagon 1 Washington, D. C. 20310
102	Dir. of Defense Research & Engineering ATTN: Technical Library RM 3E-1039, The Pentagon 1 Washington, D. C. 20301	406	Chief of Research & Develop- ment Department of the Army 1 Washington, D. C. 20315
104	Defense Communications Agency ATTN: Code 340 1 Washington, D. C. 20305	409	Commanding General U. S. Army Materiel Command ATTN: AMCMA-EE 1 Washington, D. C. 20315
200	Chief of Naval Research ATTN: Code 427 Department of the Navy 1 Washington, D. C. 20325	415	Commanding General U. S. Army Materiel Command ATTN: RSCH, Dev & Engr. Dir. 1 Washington, D. C. 20315
205	Director U. S. Naval Research Laboratory ATTN: Code 2027 1 Washington, D. C. 20390	428	Commanding General U. S. Army Combat Development Command ATTN: CDCMS-E 1 Fort Belvoir, Va. 22060
206	Commanding Officer and Director U. S. Navy Electronics Laboratory ATTN: Library 1 San Diego, California 92152	480	Commanding Officer USASA Test & Evaluation Cen 1 Fort Huachuca, Arizona 85613
210	Commandant, Marine Corps Hq, U. S. Marine Corps ATTN: Code A04C 1 Washington, D. C. 20380	483	Commander U. S. Army Research Ofc. (Durham) Box CM-Duke Station 1 Durham, North Carolina 27706
301	Rome Air Development Center (EMTLD) ATTN: Documents Library 1 Griffiss AFB, New York 13440		Defense Communication Agency ATTN: Code T222, Dr. R. Crawford 1 Washington, D. C. 20305
307	Electronic Systems Division (ESTI) L. G. Hanscom Field 1 Bedford, Massachusetts 01730	504	Commanding General U. S. Army Materiel Command ATTN: AMCRD-R (H. Cohen) 1 Washington, D. C. 20315
403	DACS for Comm-Electronics Dept. of the Army ATTN: CETS-3 1 Washington, D. C. 20315		

Distribution List (Continued)

596 Commanding Officer  
U. S. Army Combat Developments Command  
Communications-Electronics Agency  
1 Fort Monmouth, New Jersey 07703

599 Commanding General  
The MALLARD Project  
ATTN: AMCPM-MLD-TM  
1 Fort Monmouth, New Jersey 07703

605 U. S. Army Liaison Office  
MIT-Lincoln Laboratory, Room A-210  
P.O. Box 73  
1 Lexington, Massachusetts 02173

U. S. Navy Electronics Center  
ATTN: Mr. Ray Kelly  
Code 3200  
1 San Diego, California 92152

Rome Air Development Center  
ATTN: CORS, Mr. Miles Bicklehaupt  
1 Griffiss Air Force Base, New York 13440

DCA System Engineering Facility  
ATTN: Code T202 Technical Library  
1860 Wiehle Avenue  
1 Rosten, Virginia 22090

680 Commanding General  
U. S. Army Electronics Command  
Fort Monmouth, New Jersey 07703

2 AMSEL-TD-TI

1 AMSEL-RD-MT

15 AMSEL-NL-R-2

1 AMSEL-NL-R-5

UNCLASSIFIED

Security Classification

## DOCUMENT CONTROL DATA - R &amp; D

*(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)*

1. ORIGINATING ACTIVITY (Corporate author) Coordinated Science Laboratory University of Illinois at Urbana-Champaign Urbana, Illinois 61801		2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED	
		2b. GROUP	
3. REPORT TITLE  Analytic Mathematical Models of Tactical Military Communications Channels			
4. DESCRIPTIVE NOTES (Type of report and inclusive dates) Quarterly Report 1 July 1972 - 30 September 1972			
5. AUTHOR(S) (First name, middle initial, last name)  Professors Robert T. Chien, A. H. Haddad, and C. L. Chen			
6. REPORT DATE January 1973		7a. TOTAL NO. OF PAGES 14	7b. NO. OF REFS 4
8a. CONTRACT OR GRANT NO. DAAB07-71-C-0292		9a. ORIGINATOR'S REPORT NUMBER(S) ECOM-0292-5	
b. PROJECT NO. IS6.62703.A327			
c. .06		9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)	
d. .071			
10. DISTRIBUTION STATEMENT Distribution limited to US Government Agencies only; Test and Evaluation; Jan. 4, 1973. Other request for this document must be referred to Commander, US Army Electronics Command, ATTN: AMSEL-NO-R-2, Fort Monmouth, New Jersey 07703			
11. SUPPLEMENTARY NOTES		12. SPONSORING MILITARY ACTIVITY US Army Electronics Command ATTN: AMSEL-NL-R-2 Fort Monmouth, New Jersey 07703	
13. ABSTRACT  Preliminary comparison of the burst data to the model parameters obtained from the gap distributions were performed for K=2. Definite statements concerning the validity of the model may be made only after the comparison for several values of K has been performed.  Product codes were investigated and theoretical performance equations were derived.			

DD FORM 1473

UNCLASSIFIED

Security Classification



14

## KEY WORDS

## LINK A

## LINK B

## LINK C

ROLE

WT

ROLE

WT

ROLE

WT

Digital Communications

Error Correcting Codes

Channel Modeling